ME 314 - Engineering Design : Mechanical Components
Lecture 14

Note Title

5.3 Fracture Mechanics

The failure theories we discussed so far are based on the assumption that the material is perfectly homogeneous and isotropic, and thus free of defects, microscopic flaws, or cracks. However, these stress raisers normally exist on the surface or within materials. The main objective in fracture mechanics is to determine the critical crack length that will make the part fail.

For sharp cracks, or flaws and defects of small radius, the concept of stress-concentration factor does not lead to a criterion for design.

Recall the expression we obtained for σ_{max} for a plate with an elliptical hole under tension:

$$\sigma_{max} = K_t \, \sigma_{nom} \,,$$

with $K_t = 1 + 2a/c$.

Note that as "c" approaches zero, σ_{max} approaches infinity.

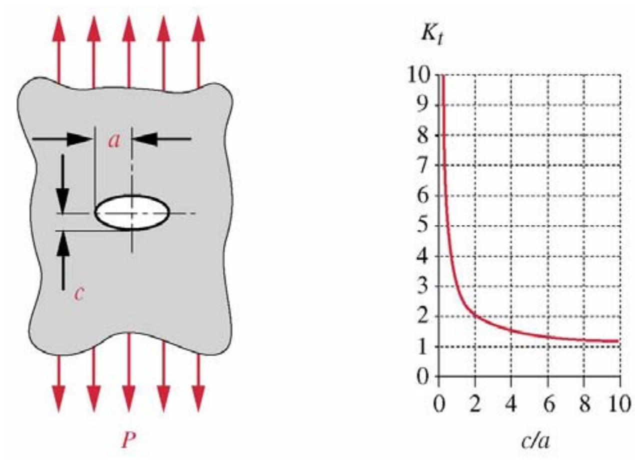


Figure 4-35
Stress Concentration at the Edge of an Elliptical Hole in a Plate.

However, on a microscopic level, an infinitely sharp crack is a hypothetical abstraction that is physically impossible; and in a highly localized deformation zone that occurs at the crack tip, the stress is finite. When the crack is of sufficient size and the stress is high enough, brittle and even ductile materials can fail in a sudden brittle fasion at stresses well below the ultimate strength.

The foundation of fracture mechanics was first established by Griffith (1921) who calculated the rate of energy for crack growth for brittle materials.

It is common for cracks to occur in welded structure, bridges, ships, aircrafts, pressure vessels, etc. Many catastrophic failures occured during WWII. The hull material of the ship shown in Fig. 5-15 was welded ductile steel and the ship had not been (dynamically) loaded significantly. In particular, the nominal stresses were well below the material's yield strength.

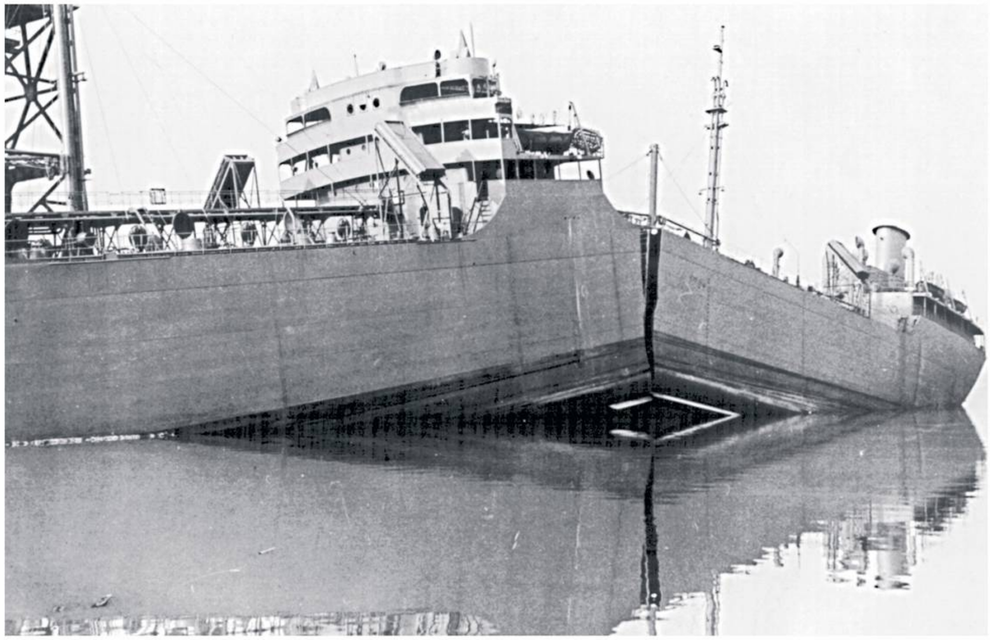


Figure 5-15
WW II Tanker Cracked in Two While Berthed Prior to Being Placed in Service, Portland, Oregon, January 16, 1943 *(Courtesy of the Ship Structures Committee, U. S. Government)*.

The 22-ft-dia rocket motor-case failed while being tested at a pressure of 542 psi. It was designed to stand 960 psi.

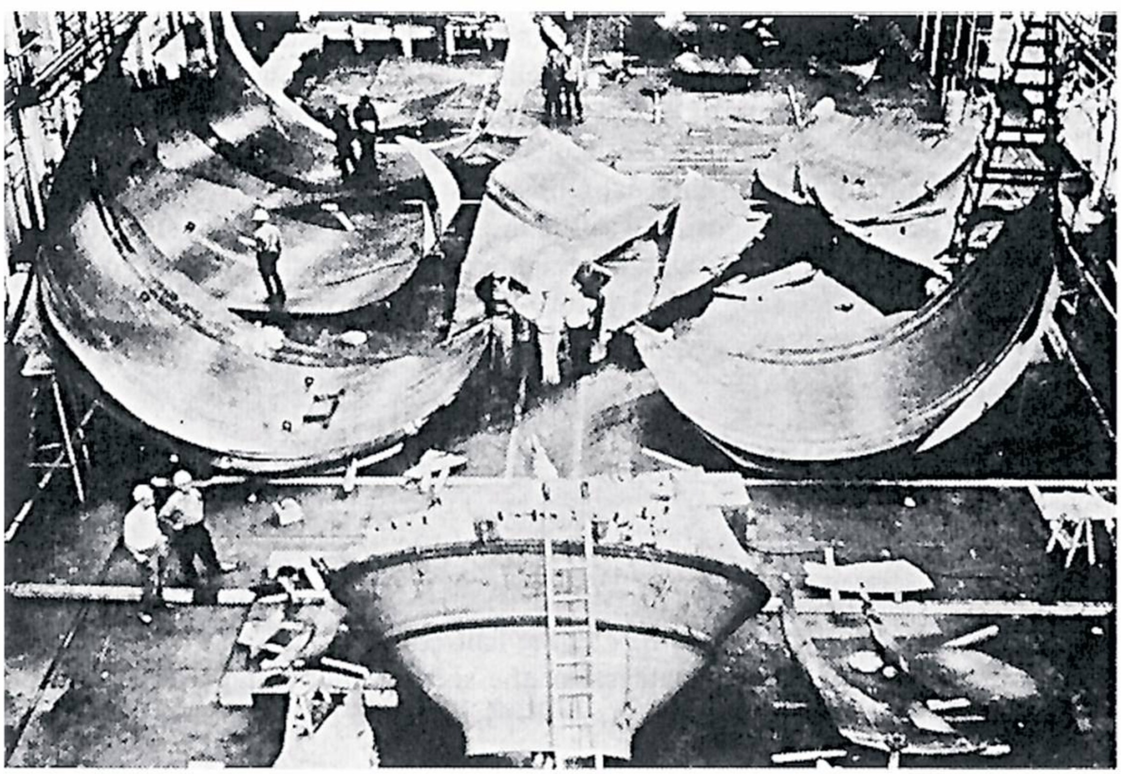


Figure 5-16
Failed Rocket-Motor Case *(Courtesy of NASA-Lewis Research Center)*.

Many bridges, aircrafts, and other structures in use today contain structural cracks. For this reason, there are regulations for periodic inspections to decide whether to replace or repair the flawed parts. These decisions are made using fracture mechanics.

Linear Elastic Fracture Mechanics (LEFM)

LEFM is based on the assumption that *a*) a crack is present, and *b*) the zone of highly localized deformation (yielding) around the crack is small compared to the dimensions of the part. Under these conditions, the bulk of the material is assumed to behave according to the Hook's law and the stress state is taken to be one of plane strain or plane stress.

Modes of Crack Displacement

Depending on the relative orientation of the loading and the crack, the applied loads may tend to pull the crack open in tension (Mode I), shear the crack in-plane (Mode II), or shear (tear) it out-of-plane (Mode III):

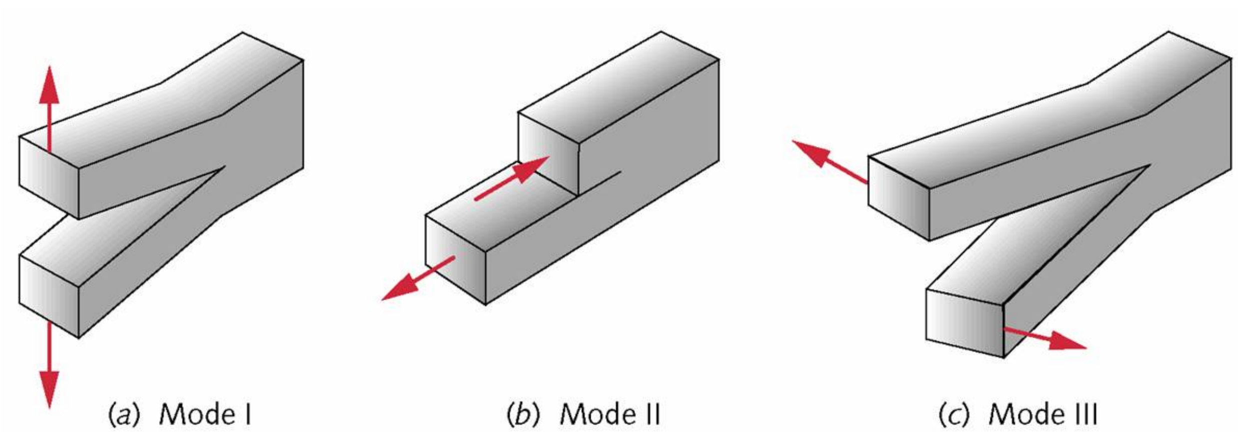


Figure 5-17
Three Modes of Crack Displacement.

Most of the research and testing has been devoted to Mode I, and the discussion here is limited to it as well.

Stress Intensity Factor K

A plate of width 2b under tension with a crack of width 2a in the center is shown in the figure (not to scale as b>>a).

From the theory of elasticity, the stresses around the crack tip are given by Eqs. (5.13) of text. They are of the form:

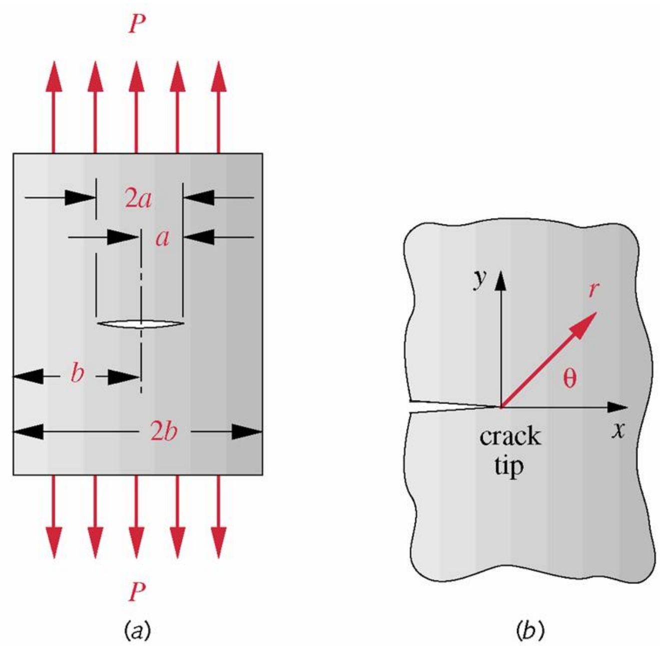


Figure 5-18
A Through-Crack in a Plate in Tension.

is called the stress intensity factor, where (r,θ) are the polar coordinates (Fig. 5.18b), and where σ is the stress in the absence of crack. Approximate relations for the three functions $f_1(\theta)$, $f_2(\theta)$, and $f_3(\theta)$ are given explicitely in Eq. (5.13) of text. The accuracy is within 10% for $a/b \leq 0.4$.

Note that at r = 0, the stresses σ_x, σ_y, and τ_{xy} are infinite. However, these stresses diminish rapidly as r increases. Note also that the units of K are either MPa-√m or kpsi-√in.

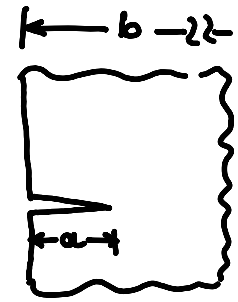
The stress intensity factor is a function of geometry, size and shape of the crack, and type of loading. For various load and geometric configurations, an additional factor, β, is introduced in (5.14a) (e.g., when "a" is not small as compared to "b"):

$$K = \beta\,\sigma\sqrt{\pi a} \qquad (5.14b)$$

Values of β can be found in handbooks. For the cracked plate in Fig. (5.18) above:

Note that β approaches 1 when a << b.

If the crack is at the edge of the plate, then



$$K = 1.12\, \sigma\sqrt{\pi a} \quad , \quad a \ll b \qquad (5.14d)$$

Accuracy of this equation is within 10% if $a/b \leq 0.13$.

Other situations are discussed on page 269 of text.

Fracture Toughness,

When the stress intensity factor, K reaches a critical value called the fracture toughness K_c, the crack will propagate suddenly to failure. Fracture toughness K_c is a material property (subject to some geometrical restrictions on the plate thickness and crack length) just like the strength and modulus of elasticity

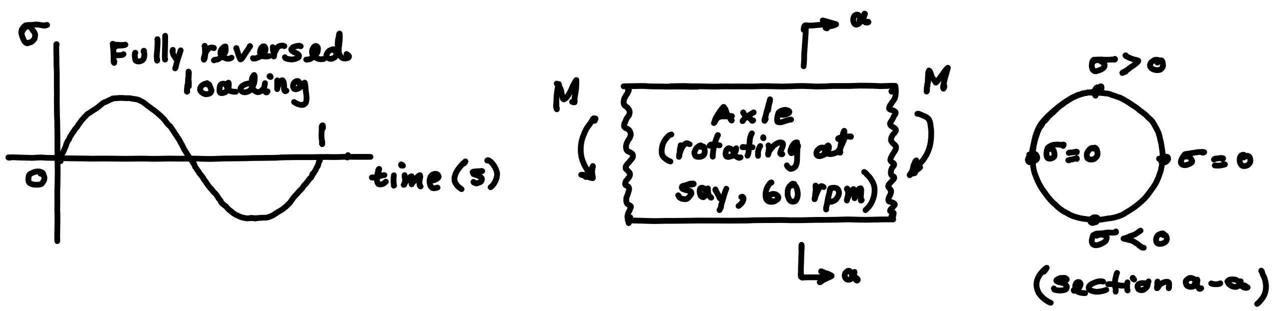
The factor of safety for fracture mechanics failure is defined as:

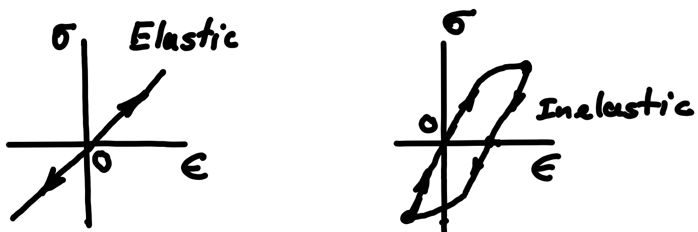
values of K_c for selected materials are given in Table 5-1 on page 271.

Example: A long plate, 100 mm-wide and 20 mm-thick is loaded in tension. A Mode-I crack 15 mm long is present in the middle of the plate. If the material is a steel alloy with S_y = 1515 MPa and K_c = 60 MPa-$\sqrt{m}$ what axial load is possible?

Chapter 6 - Fatigue Failure Theories

All the failure theories we have considered so far apply when the loading is static, i.e., the load is applied gradually so an equilibrium state is reached at every time step because there is enough time for strain to develop. Most failures in machinery, however, are due to time-varying (or dynamic) loads rather than static loads. The failure under such variable loads can occur at stress levels that are way below the yield strength of material. This is only because the stresses have been repeated a very large number of times. This type of failure which is referred to as fatigue failure was first noticed in 1800's when railroad-car axles began failing after only limited time in service. Although they were made of ductile steel, they suddenly failed in a brittle manner. These axles were fixed to the wheels and turned with them. Thus the bending stress at any point on the surface of the axle varied cyclically from positive to negative as shown:





A loading - unloading cycle (N=1)

The fibers on the surface of the axle are stressed in tension and compression 60 times each minute if the axle rotates at 60 rpm.

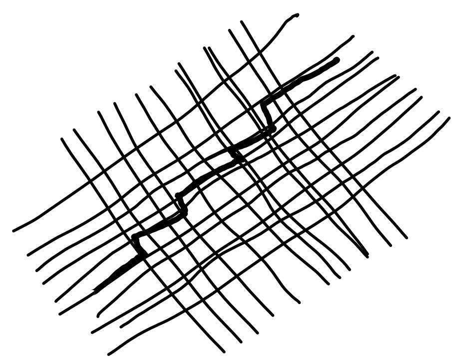
6.1 Mechanism of Fatigue Failure

The exact mechanism of fatigue failure is still a topic of research and if you happen to work in this area, you should keep up with the latest research findings and available codes (an appreciation for life-long learning which is one of our program outcomes will really pay off in this case).

An important finding so far is that fatigue failures always begin at a crack. The crack may have already been present, or it may have developed due to cyclic loading. Fatigue cracks, generally start at a stress concentration such as a notch, etc. Once the crack is initiated it propagates under the action of load application and removal until the cross-section is reduced to the point where the remaining area can no longer sustain the load, and this remaining section cracks suddenly. Hence, there are three stages of fatigue failure: Crack initiation, crack propagation, and fracture.

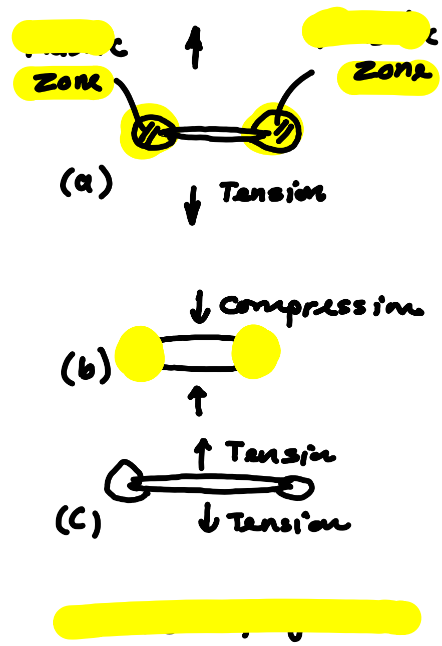
Crack Initiation Stage

When a component is subjected to cyclic loading, the oscillating stresses at a notch or a stress raiser cause localized plastic deformation at these sites, even though the stress in the section is well below the yield strength. This causes highly localized shearing along crystallographic planes or along (crystal) grain boundaries of the material. These shearing planes are referred to as slip bands or shear bands. As the stress cycles, slip bands coalesce into microscopic cracks (<0.010 in). Materials with less ductility, do not yield as much and will develop cracks more rapidly. In this sense, they are more notch sensitive. Brittle materials do not yield and may skip the initiation stage altogether.



Steps between slip bands forming cracks

Crack Propagation Stage



Once microcracks are formed, the mechanisms of fracture we discussed in Secion 5.3 (in this lecture and on page 265) become operable.

The sharp crack creates stress concentration and a plastic zone develops at the crack tip (Fig. a). Each time a tensile stress opens the crack tip, it blunts its tip and reduces the stress concentration. The crack grows by a small amount. When the stress cycles to compression, to zero, or to a lower tensile stress, the crack closes, the yielding momentarily ceases, and the crack becomes sharp, but now at its longer dimension (Fig. c). This process continues with crack growing due to tensile stress along planes normal to the maximum tensile stress (even though shear stress starts the process in ductile materials as discussed above in the crack initiation stage). The crack growth rate is very small (10^{-5} to 10^{-4} in/cycle) but this adds up over a large number of cycles. If the part is cyclically stressed in a corrosive environment the crack will grow more rapidly. This is called corrosion fatigue.

Fracture Stage

The crack growth will continue as long as cyclical tensile stress and/or a corrosive environment of sufficient severity are present. At some point, the crack size becomes large enough to raise the stress intensify factor K at the crack tip to the level of material's fracture toughness K_c and sudden failure instantaneously on the next tensile stress. cycle, i.e.,

$$K = \sigma\sqrt{\pi a} \qquad (5.14)$$

and "a" increases until $K = K_c$ at which time fracture occurs.

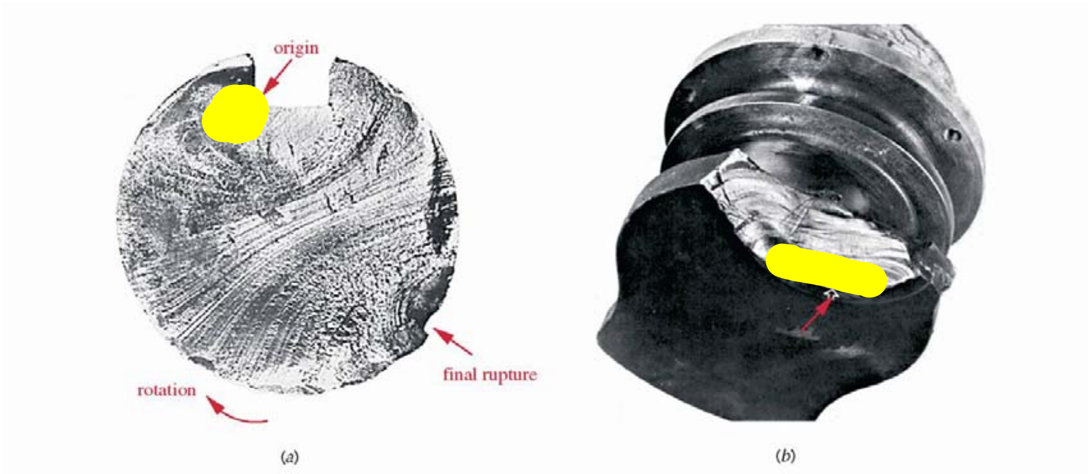


Figure 6-4

Two Parts That Failed in Fatigue. Note the Beachmarks: (*a*) 1040 Steel Keyed Shaft Failed in Rotating Bending. Crack Started at Keyway. (*b*) Diesel-Engine Crankshaft Failed in Combined Bending and Torsion. Crack Started at Arrow. (Source: D. J. Wulpi, *Understanding How Components Fail.* Am. Soc. for Metals: Metals Park, Ohio, 1990, Fig. 22, p. 149, and Fig. 25, p. 152.).

Examination with the naked eye of parts failed by fatigue loading show a typical pattern. There is the site of the original microcrack that appears burnished and often shows beachmarks (so-called because they resemble ripples left on sand by retreating waves) and a separate region that appears dull and rough, looking like a brittle fracture.

Fatigue Regimes

A component that is subjected to cyclic loading and fatigue failure is designed based on the number of cycles (of stress or strain) that it is expected to undergo in its lifetime. If the number of cycles to failure is about 1000 cycles or less, the component is said to operate in a low-cycle fatigue (LCF) regime. If the life expectancy of the compment is more than about 1000 cycles, the component is said to be in a high-cycle fatigue (HCF) regime. Note that various texts suggest slightly different dividing line between the two regimes.

6.2 Fatigue Failure Models

There are three fatigue failure models in use today. They are:

1. **The Stress-Life (S-N) approach**

2. **The Strain-Life (Є-N) approach**

3. **The Linear-Elastic, Fracture Mechanic (LEFM) approach**

The Stress-Life (S-N) Approach

In this approach, the goal is to keep local stresses in notches so low that the crack initiation stage never begins. In other words, the design goal is that stresses and strains everywhere remain in the elastic region. For this reason, this approach is most suitable for high-cycle fatigue (HCF) applications and situations where we wish to design the part for infinite life. For the same reason, however, it is the most empirical and least accurate of the three models in terms of taking into account the true local stress (or strain) states in the part, especially for low-cycle fatigue (LCF) finite-life situations where the total number of cycles is less than 1000 and the stresses are high enough to cause yielding.

The Strain-Life (Є-N) Approach

The (S-N) approach cannot model crack initiation which involves yielding. The (Є-N) approach, on the other hand, is most suitable for crack initiation stage and design for finite life in the low-cycle fatigue. It can also be used for estimating cumulative damage due to variations in the cyclic load (e.g., overloads) and creep effects due to high temperature. If is the most complicated of the three approaches and requires computation.

The LEFM Approach

This is the best approach for the crack propagation stage and design for the finite life and the low-cycle fatigue (LCF) problems.

It is often used to predict the remaining life of cracked parts in service in conjunction with nondestructive testing (NDT) in a periodic service-inspection program, especially in aerospace industry.

The approach relies on Eq. (5.14b):

$$K = \beta\, \sigma \sqrt{\pi a} \qquad (5.14b)$$

but its accuracy depends on the estimate of initial crack size "a" and the geometry factor β.

6.3 Machine-Design Considerations

Our focus in this chapter is on the stress-life (S-N) approach because it is applicable to a large class of rotating machinery whose lives are usually in the HCF range.

As an example, consider the automated production machine whose driveshaft speed is 100 rpm. In one 8-hr day, it turns (100 rpm) (60 min/hr)(8 hr /day) = 48000 rev/day. In a 260-day work-year, it turns (48000) (260)= 12.5 X 10^6 rev/ year. Hence, the driveshaft operates in the HCF regime. Please read the text for other examples.

6.4 Fatigue Loads

The character of variable loads causing fatigue failure varies substantially from one application to another. The shape of the load-time (or stress-time) function experienced by rotating machines can be modeled as shown in Figure 6-6.

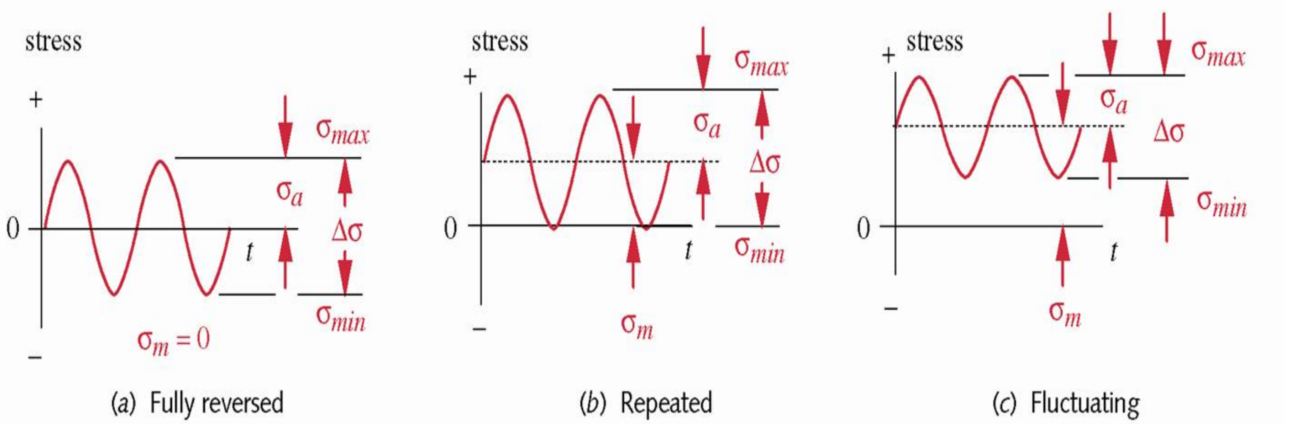


Figure 6-6

Alternating, Mean, and Range Values for Fully Reversed, Repeated, and Fluctuating Cyclic Stresses.

Note that the shape of stress-time function in other equipment may be quite different but fatigue failure does not depend on the waveform of the load-time function.

Some terminology:

Stress Range:

Alternating Component:

Mean Component:

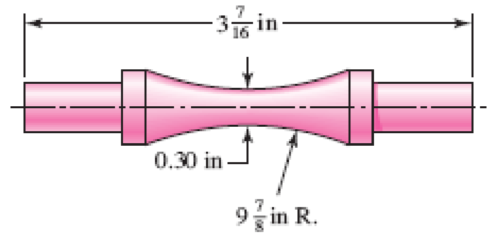
Stress Ratio:

Amplitude Ratio:

For fully reversed:

6.5 & 6.6 Measuring Fatigue Failure Strength

Most of the fatigue-strength data is for a rotating beam in fully reversed bending which were obtained by R.R. Moore using a highly polished specimen rotating at 1725 rpm. The test is run at one stress level until specimen fails.

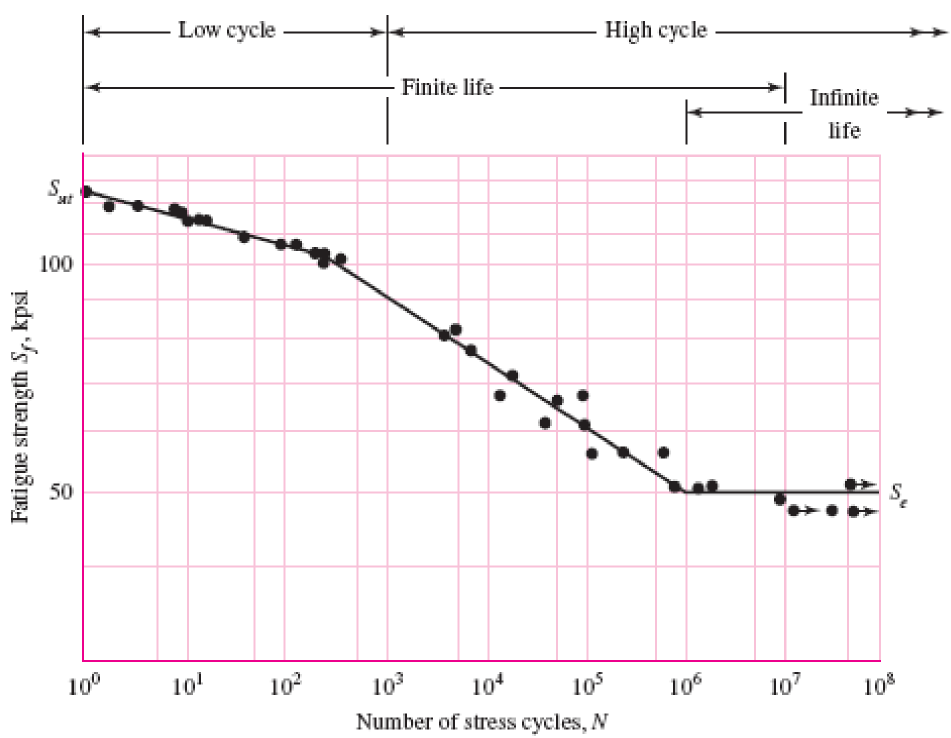


~~Figure 6-9~~

Test-specimen geometry for the R. R. Moore rotating-beam machine. The bending moment is uniform over the curved at the highest-stressed portion, a valid test of material, whereas a fracture elsewhere (not at the highest-stress level) is grounds for suspicion of material flaw.

The number of cycles to failure and the stress level are recorded for each test and are plotted in the S-N diagram shown below. Note that it takes half a day to reach a million cycles and about 40 days to reach 100 million cycles.

In the first test, stress is about S_{ut} and the specimen fails in the 1st cycle. The second test is just under S_{ut} and so on. Given the applied loads on a component, we can compute the applied stresses and then using the S-N diagram, we can find the life, i.e., the number of cycles N to failure.



An S-N diagram for steel with S_{ut} = 116 kpsi

The fatigue strength falls steadily and linearly (on log-log coordinates) as a function of N until reaching a knee at about 10^6 to 10^7 cycles. This knee defines an endurance limit $S_{e'}$ for the material that the specimen is made of.

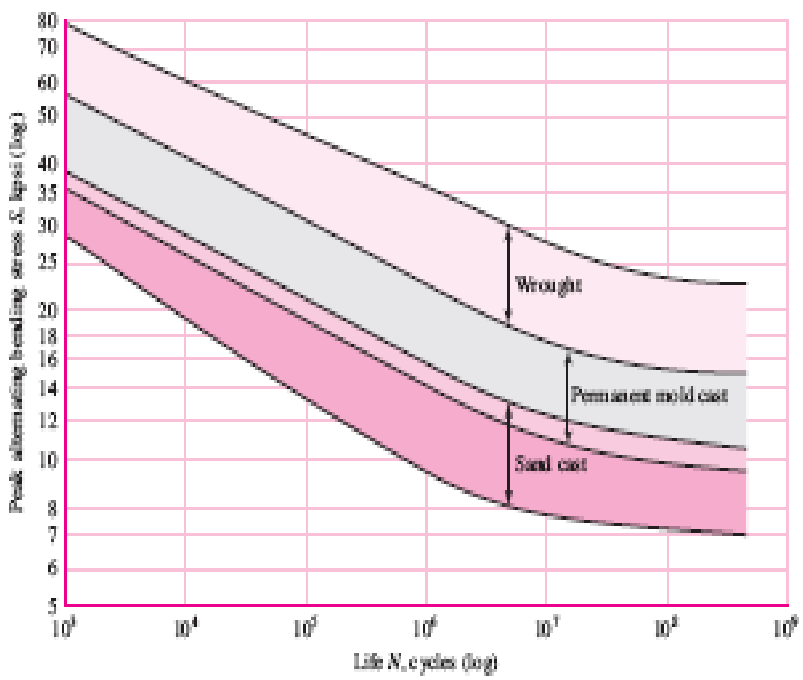
Hence, the endurance limit is a stress level below which the specimen can be cycled infinitely without failure.

Not all materials exhibit this knee and the term endurance limit is used only for those that have one.

For those materials that do not have an endurance limit, e.g., aluminum, a fatigue strength, S_f, can be defined at any N (usually, N= 5 x 10^8 cycles).

Figure 6–11

SN bands for representative aluminum alloys, excluding wrought alloys with S_{ut} < 38 kpsi. (From R. C. Juvinall, *Engineering Considerations of Stress, Strain and Strength.* Copyright © 1967 by The McGraw-Hill Companies, Inc. Reprinted by permission.)



An important question is whether or not the endurance limit, $S_{e'}$ is correlated with the ultimate tensile strength, S_{ut}.